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ORIGINAL ARTICLE

Competition, agglomeration, and tenant composition in shopping malls

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Abstract

Previous models of tenant composition in shopping malls have focused on traditional anchor and nonanchor retailers who sell similar merchandise. With the changing preferences of modern shoppers who seek unique and entertaining experiences, this article introduces a new type of store known as "specialty stores" that offer experiential consumption. Using a dynamic game model that considers the trade-off between the benefits of agglomeration and the costs of competition, we re-examine the tenant optimization problem faced by mall owners in the current retail environment. Our findings show that specialty stores have a significant impact on the optimal tenant mix and the rent revenue of developers. This article provides valuable insights into the optimal tenant composition for large-scale shopping centers that cater to contemporary consumers.

KEYWORDS

agglomeration and competition, anchor stores, rents, retail, shopping malls, tenant composition

1 | INTRODUCTION

The retail industry in the United States has undergone enormous restructuring, resulting in the construction of new retail space, abandonment of nearby space, bankruptcies, mergers, and acquisitions. Many large retail operators are divesting underperforming properties, and once-successful retailers such as Macy's and JCPenney are now striving for survival. Prior theoretical studies on tenant mix focus on the collocation benefits among stores, especially the anchor and retail stores (e.g., Stahl, 1982, Brueckner, 1993, Benjamin et al., 1992, Konishi & Sandfort, 2003). In these models, the traditional anchor stores (e.g., Nordstrom and Macy's) attract high volumes of customers, and smaller retailers, such as apparel stores, pay a higher rent to locate inside the mall. However, US retail has transformed enormously over the past two decades. Nowadays, mall owners have introduced "experiential" stores that offer differentiated amenities not easily found online and thus do not compete with these traditional brick-and-mortar retailers to attract physical store foot traffic. Therefore, a new model is necessary to analyze the tenant mix and account for this emerging trend with the experiential concept. This article attempts to fill this void by introducing a new model that incorporates experiential stores and analyzes their impacts on the optimal tenant mix for shopping malls. By doing so, we provide insight into the evolving retail environment and assist mall owners in remaining competitive in the highly dynamic retail industry.

We build a model to analyze the optimal tenant mix of three types of stores: an anchor store, nonanchor retail stores, and specialty stores. The anchor store has a reputation and pricing advantage, attracting consumers to the mall with competitive prices and a wide variety of commodities.[1] Nonanchor stores, while competing with the anchor, offer different product choices to consumers and benefit from the anchor's reputation. While previous studies have focused on the collocation benefits between anchors and nonanchor tenants, we introduce a new type of tenant—specialty stores. These include experiential retailers, restaurants, health clubs, entertainment, and other nontraditional tenants. A key feature of the specialty stores is that they do not compete with the anchor and are becoming more prevalent in malls to increase foot traffic.

Our model incorporates the trade-off between agglomeration benefits and the competition costs among the three types of tenants. With a well-known brand name and pricing advantage, the anchor store plays an important role in attracting consumers to the mall. As the anchor store provides positive externalities by increasing consumer traffic, it receives rent subsidies from mall owners (Pashigian & Gould, 1998, Gould et al., 2005). However, the anchor tenant and retail stores are competitors since they offer comparable substitutes to consumers (Konishi and Sandfort, 2003), resulting in part of the anchor's profit being diluted by the presence of nonanchor retailers. Offering tenancies to more nonanchor retail stores can be an effective strategy to attract more consumers. However, this comes at the expense of lower rent revenue since having more retailers increases competition, making nonanchor retail tenants less willing to pay high rents. This is where specialty stores play a crucial role by offering complementary products and services that attract consumers. The competition between the anchor and retail stores (Konishi and Sandfort, 2003, Konishi, 2005) highlights the importance of tenanting specialty stores in the shopping mall, especially when the anchor store's traffic-drawing power is significantly reduced. In our model, the anchor tenant drives consumer traffic because of its lower product prices, while specialty stores complement the anchor and attract consumers with differentiated amenities that are not easily transferable to the Internet.

Our approach considers the simultaneous creation of agglomeration and competition within a cluster of stores in a mall. Consumers decide to visit the mall if their utility from visiting is greater than their commuting costs (Monden et al., 2021). The presence of more stores offers a wider range of commodity choices to consumers, leading to increased consumer traffic and generating externalities consistent with a large body of literature on agglomeration economies (Koster et al.,

[1] Prior studies suggest that the presence of anchor stores increases mall traffic by attracting shoppers who do not know their purchasing preferences. Lesser-known nonanchor tenants can thus free-ride off the reputation of well-known anchor stores. Thus, the nonanchor retailers are willing to pay higher rents by locating closer to the anchors.

2019, Melo et al., 2009, and Combes et al., 2008). However, the cluster of stores can also lead to increased competition among tenants, negatively affecting their profits. As a result, the mall owner must consider all these trade-offs when determining the optimal tenant composition. Our model solves the optimization problem for a representative mall developer to maximize profit by balancing the agglomeration and competition effects.

Our study finds that specialty stores provide a greater marginal increase in rent revenue for developers than retail stores due to their complementary role, which does not reduce the profitability of the anchor store or its ability to draw traffic. The equilibrium composition of tenants in the mall is mainly determined by the tenant characteristics such as product substitution, marginal production costs, and reservation values. Among these factors, changes in marginal production costs and reservation values of specialty stores have the most significant impact on the developer's equilibrium rent revenue. On the other hand, the effect of product substitution of specialty goods is the least significant. This is because specialty stores offer products that do not compete with those of the anchor store or nonanchor retailers, and thus, the competition effect only erodes the profits of specialty stores. Our findings highlight the importance of considering tenant characteristics when determining the optimal tenant mix to balance the trade-offs between agglomeration benefits and competition costs.

This article provides valuable insights into the future of the retail sector and contributes to the literature. To the best of our knowledge, this article is the first study on collocation incentives among three types of tenants: anchor, retail, and specialty stores. In a seminal work, Konishi and Sandfort (2003) studied the store collocation problem between anchors and nonanchor tenants. Our model differs in several ways from Konishi and Sandfort (2003) and other prior studies. First, by considering the shifting preferences toward experiential shopping, we introduce a new type of tenant—specialty store. Specialty stores play a complementary role in attracting consumers, and their commodities are not substitutes for commodities sold by the anchor and nonanchor retail stores. Tenanting specialty stores in a shopping mall is increasingly popular since the anchor store can no longer benefit consumers with preference uncertainty. Preference uncertainty has been an important assumption in most of the prior literature.[2] In contrast, our model allows perfect price information due to the prevalence of digital marketing. Second, in Konishi and Sandfort's model, it is assumed that competition is driven by consumers who buy at most one unit of the commodity, either at the anchor store or at one of the retailers. Instead, our model allows consumers to purchase multiple goods and products, and the competition comes from store clustering rather than individual consumer choices. Our model considers different features of store types in the tenant mix problem, offering the flexibility to add more product types, whether substitutes or not, and demonstrating consumer behavior towards different product types with perfect information on commodities.

Our study highlights the importance of specialty stores during the ongoing revolution of the retail industry. As online shopping continues to grow, mall owners are shifting focus to specialty stores, such as full-service restaurants, movie theaters, and active entertainment like skating rinks to offer unique experiences and create lifestyle environments. Our model sheds light on owners' optimal tenant mix, construction, destruction, and rebuilding decisions and provides valuation and investment implications for malls with different tenant compositions. In addition, there are over 1000 vacant anchor stores at US malls (Green Street, 2022). Many of these malls have

[2] Most of the prior studies conclude that anchors generate traffic to a shopping center or retail cluster because customers have uncertainty about prices and preferences. Thus, consumers can economize on transportation costs by making multipurpose shopping trips. See, for example, Stahl (1982) and Brueckner (1993).

cotenancy clauses in place, which would allow inline tenants to pay reduced rent when the anchor stores become dark and even allow them to terminate their lease without penalty if the landlord cannot fill the anchor vacancies within a given time period. Highlighting the important role of specialty stores, our results provide implications for developers and investors on vacancy-filling strategies and potential lease agreement contracts. In addition, local planners can enhance retail vitality by promoting the experiential aspects of shopping in demolition and rebuilding decisions.

The remainder of this article is organized as follows. Section 2 outlines our model. Section 3 discusses numerical examples. Section 4 concludes the article with a brief discussion of the model limitations and industry implications.

2 | MODEL

Consider a monocentric city in a featureless plain (Alonso, 1964, Mills (1972), and Muth (1969)) with a monopolist shopping mall located in the city' s center. The city is populated by a continuum of consumers uniformly distributed in a circle of radius T (the city boundary). Consumers with identical preferences ex ante travel to the mall to shop. They differ in commuting costs, measured as a consumer' s location distance from the shopping center $0 \leq t \leq T$. Consumers have preferences for various goods, and their utilities increase as the variety of goods offered by a shopping mall widens. Consumers have perfect information on the prices of different stores' goods (Arakawa, 2006), meaning there are no search costs for the consumers.

A profit-maximizing developer determines the tenant mix at her mall. In the shopping mall, there is an anchor tenant and two types of nonanchor stores in the tenant mix. Specifically, the first type of nonanchor stores, referred to as the "retail store," competes with the anchor tenant. On the other hand, the "specialty stores," which is the second type of nonanchor stores, do not compete with the anchor but rather among themselves. For this model, we assume that each nonanchor tenant sells a single type of differentiated good, denoted as R for retail stores and S for specialty stores. In contrast, the anchor store offers two distinct categories of products: one is denoted as A, which represents goods that are imperfect substitutes for the retail stores' products R; the other category is denoted as M, representing a unique product that is sold exclusively (i.e., a monopoly product) by the anchor store. The mall owner has control over the number of product categories available at nonanchor stores by restricting the number of such stores. However, the owner does not influence the number of product categories offered by the anchor store, as these large department stores typically follow a standardized product assortment. Specialty stores play a vital role in enticing consumers to visit the mall by offering experiential services such as entertainment, food and beverage options, sports, healthy care, and professional services. They complement the offerings of anchor and retail stores and provide unique goods and services (denoted as S) that are not substitutes for what is available elsewhere in the mall. However, it is worth noting that competition exists between the specialty stores themselves.

The mall owner's decision on tenant mix in this model is equivalent to determining the number of nonanchor tenants (k_R, k_S), where k_R and k_S are the number of retail and specialty stores, respectively. Since we assume that each nonanchor retail store sells only one type of differentiated product (R), we can also interpret k_R and k_S as the number of different commodities sold by retail and specialty stores. Given that the assortment at the anchor store is usually very standardized, which limits the number of commodities it can offer in the mall. Specifically, we set the maximum number of products the anchor store can offer, including both its monopoly product (M) and substitutable product (A), to a fixed value denoted as $\bar{k}_0$. When $k_R < \bar{k}_0$, the anchor store sells

k_R types of product (A) that compete with retail stores' products and $\bar{k}_0 - k_R$ types of monopoly products that do not compete with retail stores.

The tenants are assumed to sign a lease contract with the developer. The contract specifies a rent r^A, r^R, and r^S for the anchor, retail, and specialty stores, respectively. Each type of store has its reservation profits (net of rents), denoted as ρ^A, ρ^R, and ρ^S. Therefore, those stores are willing to accept the contract only when their net profits are not less than their reservation profits. The rents are the fixed cost to tenants and therefore do not affect the decisions on the prices of commodities. In equilibrium, efficient rent extraction implies that the rent payments for all tenants are the difference between their sales and reservation profits.

Both developers and tenants have perfect information on consumer preferences, and consumers have perfect information on the prices of every commodity. That is to say, there is no asymmetric information between consumers and sellers in the model.

The sequence of the game is as follows:

1. A developer decides on the rent payments and composition of anchor, retail, and specialty stores at her shopping mall.
2. Consumers decide to visit the shopping mall depending on their commute costs and utilities of visiting. This stage determines the market size or consumer traffic of the shopping mall.
3. The stores set their prices simultaneously.
4. Consumers make purchase decisions.

The solution concept is that of subgame perfect Nash equilibrium, and we derive the equilibrium by backward induction, starting from the consumer' s problem.

2.1 | Consumers

Consumers can visit three types of stores: anchor, retail, and specialty. As described above, we assume that the anchor store sells two types of products (M and A), and retail and specialty stores each sell only one type of product, R and S, respectively. For each product type, we allow a variety (e.g., shoes with different brands) indexed by i, j, z. For example, the anchor store provides i different choices for commodities M and j different choices for commodities A. In other words, the anchor store sells two types of commodities denoted by M_i and A_j. The former is a monopolized product and the latter is an imperfect substitute for the retail store commodities denoted by R_j. Both R_j and A_j share the same subscription j because they are close substitutes. For example, consumers could buy Nike shoes from either the anchor store (e.g., Macy's) or a retail store (e.g., a Nike store or a shoe store). Therefore, there is competition between the anchor and retail stores.

Moreover, we take into account the competition between various product varieties in our analysis. For instance, within the product types A and R, each variety, denoted by j and l, respectively, competes with the other. Equations (2) and (3) include the impact of other product varieties. The substitution effects of different product varieties on the A and R product types are not symmetrical. Specifically, we assume that the effect on the demand of a particular variety q_j^A is smaller than that of the demand of q_j^R, denoted by the degree of substitution ϵ being smaller than ϕ. This assumption is reasonable because, compared with small-scale retail stores, anchor stores are better at coordinating price promotions, utilizing retailer-managed inventory systems, and cooperative advertising (Walters, 1991, Mishra & Raghunathan, 2004, Huang et al., 2002).[3] The specialty stores in the mall offer a variety of service products denoted as S_z, with a total of z different choices available. Additionally, competition exists among the various product varieties offered by specialty stores. This is reflected in Equation (4), which takes into account the number of other specialty product varieties and their degree of substitution, represented by the parameter θ.

Consumers' shopping behaviors follow the standard linear demand functions where the quantity demanded decreases with price and its substitute. p_i^M, p_j^A, p_j^R, and p_z^S (q_i^M, q_j^A, q_j^R, and q_z^S) denote the corresponding prices (quantities) of those commodities. Specifically, consumers' demand functions for those commodities offered by different stores are given by the following:

$$q_i^M = 1 - p_i^M, \tag{1}$$

$$q_j^A = 1 - p_j^A - \epsilon \sum_{l \neq j}^{k_R} q_l^A - \epsilon \sum_{j}^{k_R} q_j^R, \tag{2}$$

$$q_j^R = 1 - p_j^R - \phi \sum_{l \neq j}^{k_R} q_l^R - \phi \sum_{j}^{k_R} q_j^A, \tag{3}$$

$$q_z^S = 1 - p_z^S - \theta \sum_{l \neq z}^{k_S} q_l^S, \tag{4}$$

where ϵ, ϕ, and θ range from 0 (nonsubstitutes) to 1 (perfect substitutes).

2.2 | Tenants

Considering the tenants' pricing decision, we inverse the above demand functions to obtain the set of price functions. The prices of goods are strategic variables for the tenants in the model and they make the pricing strategies simultaneously. Note that our theoretical framework differs from Konishi and Sandfort (2003) in the following way. Their model assumes consumers do not know the prices of the retail commodities before visiting a shopping mall. The consumers only know the price of the product sold by the anchor store. Instead, our model allows the prices of all stores to be commonly known to consumers before visiting the mall because they can easily access information about the mall through digital marketing nowadays.

By solving Equations (2) and (3) simultaneously, we can obtain the demand functions of A_j and R_j in terms of the price variables p^A and p^R:[4]

[3] Walters (1991) find that price promotion on one brand has a negative impact on sales of competing brands in the category. Thus, compared with multiple retail stores, which sell products under a single brand, the anchor can better coordinate price promotions among different brands, resulting in a lower substitution. Mishra and Raghunathan (2004) suggest that retailers are able to shift competition to manufacturers through retailer-managed inventory systems, in which the vendor (rather than the retailer) is responsible for the management of stock at the retailer. Relative to retail stores, the anchor has better bargaining power when dealing with vendors (Huang et al., 2002). This could also lead to a lower degree of substitution due to shifted competition.

[4] Please refer to Appendix A for the derivation of the demand functions of product types R and A.

$$q_j^A = a^A(k_R|\epsilon,\phi) - b^A(k_R|\epsilon,\phi)p_j^A + c^A(k_R|\epsilon,\phi)\sum_{l\neq j}^{k_R} p_l^A + d^A(k_R|\epsilon,\phi)\sum_{j}^{k_R} p_j^R, \tag{5}$$

$$q_j^R = a^R(k_R|\epsilon,\phi) - b^R(k_R|\epsilon,\phi)p_j^R + c^R(k_R|\epsilon,\phi)\sum_{l\neq j}^{k_R} p_l^R + d^R(k_R|\epsilon,\phi)\sum_{j}^{k_R} p_j^A, \tag{6}$$

where $a^A, b^A, c^A, d^A, a^R, b^R, c^R, d^R$ are coefficients that depend on the number of product types k_R and the degree of substitution ϵ and ϕ.

From Equation (4), the demand functions of commodities S_z can also be derived in terms of the price variables p^S:[5]

$$q_z^S = a^S(k_S|\theta) - b^S(k_S|\theta)p_z^S + c^S(k_S|\theta)\sum_{l\neq z}^{k_S} p_l^S, \tag{7}$$

where a^S, b^S, c^S are coefficients that depend on the number of product types k_S and the degree of substitution θ.

The anchor store plays a vital role in attracting customers to the mall, thanks to its wide range of merchandise (which provides a "one-stop shopping" advantage) and effective marketing strategies. This ultimately benefits the smaller, nonanchor retail stores in the mall. Meanwhile, the retail stores aim to offer consumers a more diverse selection of products. Due to economies of scale, the anchor store has a price advantage over the retail store R_j for each substitutable product j, as its marginal production cost is lower ($mc^A < mc^R$). Although our demand functions are not directly derived from consumer utility functions, they incorporate the number of retailers and specialty stores in the mall. This endogenizes the agglomeration economies in our model. Each specialty store sells different commodities S_z at prices p_z^S. We assume that there are no specific restrictions on the marginal production cost of commodity S_z, and for simplicity, we assume homogeneous marginal production costs for the same store types, where mc^M, mc^A, mc^R, and mc^S are not indexed by i, j, or z.

The total rents (r^A, r^R, r^S) are respectively written on the contracts for anchor, retail, and specialty stores. Taking the rents (r^A, r^R, r^S) as given, all tenants choose the optimal pricing strategies simultaneously.[6] The anchor chooses price p_i^M and p_j^A to maximize the profits of selling the commodities M_i and A_j :

$$\max_{p_i^M, p_j^A} (p_i^M - mc^M)q_i^M + (p_j^A - mc^A)q_j^A - r^A.$$

By substituting the demand functions (1) and (5), we can solve the optimal prices (p_i^{M*}, p_j^{A*}) as follows:

$$p_i^{M*} = \frac{1 + mc^M}{2}, \tag{8}$$

[5] Please refer to Appendix B for the derivation of the demand functions of product types S.

[6] Compared with the marginal production costs, the rental costs are fixed. The developer cannot interfere with tenants' decisions on the prices.

$$p_j^{A*} = \frac{a^A + b^A mc^A + c^A \sum_{l \neq j}^{k_R} p_l^A + d^A \sum_j^{k_R} p_j^R}{2b^A}. \tag{9}$$

To simplify the notation, we denote that $a^A = a^A(k_R|\epsilon, \phi)$, $b^A = b^A(k_R|\epsilon, \phi)$, $c^A = c^A(k_R|\epsilon, \phi)$, and $d^A = d^A(k_R|\epsilon, \phi)$.

Similarly, the retail stores choose price p_j^R to maximize the profits of selling each commodity R_j:

$$\max_{p_j^R} (p_j^R - c^R) q_j^R - r_j^R.$$

By substituting the demand function (6), we can solve the optimal prices p_j^{R*}as follows:

$$p_j^{R*} = \frac{a^R + b^R mc^R + c^R \sum_{l \neq j}^{k_R} p_l^R + d^A \sum_j^{k_R} p_j^A}{2b^R}. \tag{10}$$

To simplify the notation, we denote that $a^R = a^R(k_R|\epsilon, \phi)$, $b^R = b^R(k_R|\epsilon, \phi)$, $c^R = c^R(k_R|\epsilon, \phi)$, and $d^R = d^R(k_R|\epsilon, \phi)$.

The optimal prices p_j^{A*} and p_j^{R*} depend on the prices of other varieties of commodities A and R. By symmetry, the first-order conditions for different varieties of commodity prices p_j^A and p_j^R imply that the optimal prices $p_1^{A*} = \cdots = p_{k_R}^{A*}$ and $p_1^{R*} = \cdots = p_{k_R}^{R*}$. Therefore, we can simplify Equations (9) and (10) and then simultaneously derive the equilibrium prices (p_j^{A*}, p_j^{R*}) in the following forms:

$$p_j^{A*} = \frac{(a^A + b^A mc^A)[2b^R - c^R(k_R - 1)] + (a^R + b^R mc^R) d^A k_R}{[2b^A - c^A(k_R - 1)][2b^R - c^R(k_R - 1)] - d^A d^R k_R^2}, \tag{11}$$

$$p_j^{R*} = \frac{(a^R + b^R mc^R)[2b^A - c^A(k_R - 1)] + (a^A + b^A mc^A) d^R k_R}{[2b^A - c^A(k_R - 1)][2b^R - c^R(k_R - 1)] - d^A d^R k_R^2}. \tag{12}$$

After that, we can obtain the equilibrium quantities $(q_i^{M*}, q_j^{A*}, q_j^{R*})$ by substituting these equilibrium prices into the demand functions (1), (5), and (6). Notice that equilibrium prices (p_j^{A*}, p_j^{R*}) and quantities (q_j^{A*}, q_j^{R*}) depend on the variable k_R and the parameters (ϵ, ϕ, mc).

The specialty stores choose price p_z^S to maximize the profits of selling commodity z:

$$\max_{p_z^S} (p_z^S - mc^S) q_z^S - r_z^S.$$

By substituting the demand function (7), we can solve the optimal prices of specialty stores p_z^{S*} as follows:

$$p_z^{S*} = \frac{a^S + b^S mc^S + c^S \sum_{l \neq z}^{k_S} p_l^S}{2b^S}.$$

To simplify the notation, we denote that $a^S = a^S(k_S|\theta)$, $b^S = b^S(k_S|\theta)$ and $c^S = c^S(k_S|\theta)$.

The optimal prices p_z^{S*} depend on the prices of other varieties of commodities S. By symmetry, the first-order conditions for different varieties of commodity prices p_z^S imply that the optimal prices $p_1^{S*} = \cdots = p_{k_S}^{S*}$. Therefore, we can simplify Equation (13) and derive the equilibrium prices p_z^{S*} in the following form:

$$p_z^{S*} = \frac{a^S + b^S mc^S}{2b^S - (k_S - 1)c^S}. \tag{13}$$

Then we can obtain the equilibrium quantity q_z^{S*} by substituting the equilibrium price p_z^{S*} into the demand function (7). Notice that equilibrium prices p_z^{S*} and quantities q_z^{S*} depend on the variable k_S and the parameters (θ, mc).

2.3 | Equilibrium market size

Given the equilibrium pricing strategies of all the stores derived in the previous section, we can determine consumers' decisions about whether to visit the mall. A consumer's value from shopping at the mall can be evaluated with perfect information about the product prices. We assume that the consumer's utility from visiting the mall is a function of the consumption quantities over four types of commodities offered in the mall. Specifically, we use a standard Cobb-Douglas utility function as follows:

$$U(q_i^M, q_j^A, q_j^R, q_z^S) = (\sum_{i=1}^{\bar{k}_0 - k_R} q_i^M)^{1/4} (\sum_{j=1}^{k_R} q_j^A)^{1/4} (\sum_{j=1}^{k_R} q_j^R)^{1/4} (\sum_{z=1}^{k_S} q_z^S)^{1/4}. \tag{14}$$

It is reminded that there is a maximum number of commodities $\bar{k}_0$ offered by the anchor. By this restriction, the number of monopoly types commodities q_i^M is $\bar{k}_0 - k_R$ and the number of substitutable types commodities q_j^A is k_R.

This utility function represents a consumer's preference for quantity and product variety, consistent with Ushchev et al. (2015)'s notion that consumers prefer product variety. In equilibrium, the consumers' purchasing demand equals the commodity supplies $(q_i^{M*}, q_j^{A*}, q_j^{R*}, q_z^{S*})$ from the stores. A necessary and sufficient condition for a consumer visiting a mall requires that the value of visiting must be no less than the commuting costs t. We assume that consumers are uniformly within a circle with a radius of T, representing the maximum commuting costs incurred. Hence, only consumers whose commuting costs t are no greater than the equilibrium value $U^* = U(q_i^{M*}, q_j^{A*}, q_j^{R*}, q_z^{S*})$ will visit the mall. As transportation costs increase with distance from the mall location, there will be a marginal consumer with transportation cost $t^* \equiv U^*$ who is indifferent to visiting or not. That is, the number of consumers who visit the shopping mall (the market size, or "traffic") is given by

$$\mu = \frac{\text{Area of circle of radius } t^*}{\text{Area of circle of radius } T} = \left(\frac{t^*}{T}\right)^2 = \left[\frac{U(q_i^{M*}, q_j^{A*}, q_j^{R*}, q_z^{S*})}{T}\right]^2. \tag{15}$$

Since the equilibrium quantities demanded from consumers $(q_j^{A*}, q_j^{R*}, q_z^{S*})$ depend on the variables k_R and k_S, the market size also depends on these variables such that $\mu = \mu(k_R, k_S)$.

2.4 | Equilibrium profits

We can calculate the equilibrium profits for anchor, retail, and specialty stores with the equilibrium quantities, prices, and market size. Let π be the profit of each store from one consumer and μ be the total number of consumers (market size), so $\mu\pi$ is the total profit of each store. Π is the profit net of rent. The total profits of the anchor tenant are composed of two different types of commodities: M_i, the unique product, and A_j, the imperfect substitute for the retail store products. The anchor store's equilibrium net profit is given by

$$\begin{aligned}\Pi^{A*} &= \mu\pi^{A*} - r^A \\ &= \mu(k_R, k_S)\left[\sum_{i=1}^{\bar{k}_0-k_R}(p_i^{M*} - mc^M)q_i^{M*} + \sum_{j=1}^{k_R}(p_j^{A*}(k_R) - mc^A)q_j^{A*}(k_R)\right] - r^A. \end{aligned} \tag{16}$$

Similarly, we have the following profit functions for the retail and specialty stores. The equilibrium net profit of each retail store selling commodity R_j is given by

$$\begin{aligned}\Pi_j^{R*} &= \mu\pi_j^{R*} - r_j^R \\ &= \mu(k_R, k_S)(p_j^{R*}(k_R) - mc^R)q_j^{R*}(k_R) - r_j^R. \end{aligned} \tag{17}$$

The equilibrium net profit of each specialty store selling commodity S_z is given by

$$\begin{aligned}\Pi_z^{S*} &= \mu\pi_z^{S*} - r_z^S \\ &= \mu(k_R, k_S)(p_z^{S*}(k_S) - mc^S)q_z^{S*}(k_S) - r_z^S. \end{aligned} \tag{18}$$

2.5 | Developer's tenant mix decision

Given the profitability of each type of store, we can analyze the optimal tenant composition. All the stores are supposed to have outside profit opportunities. Anchor, retail, and specialty tenants will only accept lease offers if their net profits exceed their contract or reservation values (ρ^A, ρ^R, and ρ^S), which are determined by the market. The mall developer selects the number of retail (k_R) and specialty (k_S) stores while ensuring that the total number of stores does not exceed the mall's maximum capacity ($\bar{k}$). The level of agglomeration economies increases as the number of products in the mall grows, but more substitutes lead to greater competition and lower profits for all stores. The developer, who aims to maximize the rental income from her tenants, must choose the optimal tenant composition by considering the trade-off between agglomeration economies and competition.

The developer's total rent revenue from any tenant composition is defined as $\Pi^D(k_R, k_S)$ such that

$$\Pi^D(k_R, k_S) = r^A(k_R, k_S) + \sum_{j=1}^{k_R} r_j^R(k_R, k_S) + \sum_{z=1}^{k_S} r_z^S(k_R, k_S), \tag{19}$$

where r^A, r_j^R, and r_z^S are the effective rents charged to the anchor, retail, and specialty stores, respectively. These rents are set to ensure that the net profits of each store type, after rent payment, equal their corresponding reservation profits. Therefore, the equilibrium rental income of each store type is the difference between their equilibrium profit and reservation profit:

$$r^A(k_R^*, k_S^*) = \mu(k_R^*, k_S^*)\pi^{A*}(k_R^*) - \rho^A, \tag{20}$$

$$r_j^R(k_R^*, k_S^*) = \mu(k_R^*, k_S^*)\pi_j^{R*}(k_R^*) - \rho^R, \tag{21}$$

$$r_z^S(k_R^*, k_S^*) = \mu(k_R^*, k_S^*)\pi_z^{S*}(k_S^*) - \rho^S. \tag{22}$$

A profit-maximizing developer determines the tenant mix by choosing the combination of retail and specialty stores from the feasible set $\mathcal{F} = \{(k_R, k_S) \in \{0, 1, \ldots, \bar{k}\} \times \{0, 1, \ldots, \bar{k}\} | k_R + k_S \leq \bar{k}\}$ to maximize total rental income:

$$(k_R^*, k_S^*) = \arg \max_{(k_R, k_S) \in \mathcal{F}} \Pi^D(k_R, k_S) \tag{23}$$

subject to

$$k_R^* + k_S^* \leq \bar{k}.$$

This optimal tenant composition (k_R^*, k_S^*) is the subgame perfect equilibrium.

Although a closed-form solution for (k_R^*, k_S^*) is not available, the trade-off of allocating a retail or specialty store can be derived analytically.[7] We can evaluate the impact of adding another retail and specialty store to the developer's profit by taking the derivative of Π^D with respect to k_R and k_S, respectively:

$$\begin{aligned} \frac{d\Pi^D}{dk_R} &= \frac{d}{dk_R} r^A(k_R, k_S) + \frac{d}{dk_R} \sum_{j=1}^{k_R} r_j^R(k_R, k_S) + \frac{d}{dk_R} \sum_{z=1}^{k_S} r_z^S(k_R, k_S) \\ &= \underbrace{r^R}_{\text{rent income}} + \underbrace{\left[\pi^A + k_R \pi^R + (\bar{k} - k_R)\pi^S\right] \frac{d\mu}{dk_R}}_{\text{agglomeration effect}} + \underbrace{\mu\left(\frac{d\pi^A}{dk_R} + k_R \frac{d\pi^R}{dk_R} + (\bar{k} - k_R)\frac{d\pi^S}{dk_R}\right)}_{\text{competition effect}} - \underbrace{r^S}_{\text{opportunity cost}}, \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{d\Pi^D}{dk_S} &= \frac{d}{dk_S} r^A(k_R, k_S) + \frac{d}{dk_S} \sum_{j=1}^{k_R} r_j^R(k_R, k_S) + \frac{d}{dk_S} \sum_{z=1}^{k_S} r_z^S(k_R, k_S) \\ &= \underbrace{r^S}_{\text{rent income}} + \underbrace{\left[\pi^A + (\bar{k} - k_S)\pi^R + k_S \pi^S\right] \frac{d\mu}{dk_S}}_{\text{agglomeration effect}} + \underbrace{\mu\left[\frac{d\pi^A}{dk_S} + (\bar{k} - k_S)\frac{d\pi^R}{dk_S} + k_S \frac{d\pi^S}{dk_S}\right]}_{\text{competition effect}} - \underbrace{r^R}_{\text{opportunity cost}}. \end{aligned} \tag{25}$$

The expression for the total effect of adding a retail store has four main components. The first term, r^R, represents the increase in rental income from the additional retail store. The second term is the agglomeration effect, which captures the effect of the new store on consumer volume and the total profits of all stores. The third term is the competition effect, which measures the

[7] For more information on this proof, please refer to Appendix C

TABLE 1 Parameter definitions and baseline values.

Parameters	Description	Value*
ϵ	The product substitution effect on the demand of commodity A_j	0.005
ϕ	The product substitution effect on the demand of commodity R_j	0.01
θ	The product substitution effect on the demand of commodity S_z	0.01
mc^M	Marginal production cost of monopoly commodity M_i by the anchor	0.05
mc^A	Marginal production cost of substitutable commodity A_j by the anchor	0.0
mc^R	Marginal production cost of substitutable commodity R_j by a retail store	0.02
mc^S	Marginal production cost of commodity S_z by a specialty store	0.02
ρ^A	Reservation value of the anchor	0.1
ρ^R	Reservation value of the retail stores	0.01
ρ^S	Reservation value of the specialty stores	0.01
$\bar{k}_0$	Maximum number of monopoly and substitutable commodity types by the anchor	30
$\bar{k}$	Maximum number of nonanchor stores in the mall	30

*The parameter values used in the baseline scenario of the numerical examples (unless otherwise specified in the figures).

reduction in profits due to increased competition. The fourth term is the opportunity cost, which represents the rental income sacrificed by a specialty store. The change in retail stores also affects the number of monopolized products q^M sold by the anchor. If the monopolized products q^M are more profitable than substitutable products q^A, the sign of $\frac{d\pi^A}{dk_R}$ is negative. Additionally, the increase in retail stores leads to less competition among specialty stores. This reduces the number of specialty stores and has a positive effect on their profits, resulting in a positive sign for $\frac{d\pi^S}{dk_R}$.

We can similarly assess the effect of adding an additional specialty store on the developer's profit. The total effect involves four factors: (1) gain of rental income, (2) agglomeration effect resulting from increased consumer volume, (3) competition effect, and (4) opportunity cost. The interpretation of each term is similar to the case discussed above. In the next section, we will provide a numerical example to illustrate how the number of retail and specialty stores affects the overall rent revenue from the anchor, retail, and specialty stores.

3 | NUMERICAL EXAMPLES

It is not possible to derive a closed-form solution for the tenant mix since the developer's optimization problem is discrete. Instead, we provide numerical examples to showcase essential features of the model, such as how market size, developer profits, and rent revenues change with the number of retail and specialty stores. Consequently, we illustrate the optimal tenant mix in a shopping mall, and all key parameter definitions and baseline values are available in Table 1. In retail businesses, predetermined capacities for both the anchor store and shopping mall are typical. Our model assumes a constant maximum of $\bar{k}_0$ commodities offered by the anchor store in the baseline scenario and a fixed capacity of the shopping mall where the total number of nonanchor stores is fixed at $\bar{k}$.

The numerical examples aim to highlight the trade-off between agglomeration and competition effects by adopting specific model parameters. The first parameter set (ϵ, ϕ, and θ) measures the degree of product substitution, indicating direct tenant competition. The second parameter set (mc^M, mc^A, mc^R, and mc^S) represents the marginal production costs of commodities sold by each tenant type, showing differences in production technology among them. Lower production costs signify a stronger competitive advantage in attracting consumers and more significant agglomeration benefits. The third parameter set (ρ^A, ρ^R, and ρ^S) is the reservation values for each tenant type, indicating outside option values that determine store stay or exit decisions. Finally, the fourth parameter set, including $\bar{k}_0$ and $\bar{k}$, is the predetermined capacities of the anchor store and shopping mall.

3.1 | Optimal tenant composition

We investigate the competition among different types of commodities, denoted as A_j and R_j. While different varieties of a specific product type compete with each other, the effects of substitution on product types A and R are not identical. As described in Section 2.1, we assume that the substitution effect on the demand for commodity A_j is smaller than that for R_j, that is, $\epsilon < \phi$. Additionally, we posit that the anchor store has a cost advantage over retailers, where $mc^A < mc^R$. Moreover, the marginal production cost of commodity q^A is assumed to be lower than that of q^M due to economies of scale, that is, $mc^A < mc^M$. Consequently, lower marginal production costs lead to lower product prices, generating a larger consumer traffic volume at equilibrium. In other words, substitutable commodities provided by the anchor have a competitive advantage in creating a larger market. Furthermore, the reservation value (ρ^A, ρ^R, ρ^S) can be regarded as the second-highest contract value from another shopping center. As the anchor store is the primary driver of consumer traffic and creates a positive externality for other stores, we assume that the reservation value of an anchor store is higher than the other two types of stores.

The objective of the developer is to maximize the total rent revenue by selecting the tenant mix (k_R, k_S). To start, we analyze the impact of allocating retail stores on the market size and the rental revenue of different types of stores. Our focus is to understand the trade-off between agglomeration and competition when allocating space to retailers. We measure agglomeration by market size, which has a scale effect on the profits of all stores. On the other hand, competition arises from store clustering. For instance, if we keep the number of specialty stores k_S constant, the higher the number of retail stores, the greater the competition between the anchor and retail stores.

Figure 1a–d displays the relationship between the number of retail stores and the market size, as well as the rent revenue generated by the anchor store, specialty stores, and retail stores. In our analysis, we keep the number of specialty stores fixed at 3 and vary the number of retail stores from 1 to 27.[8]

Figure 1a demonstrates a hump-shaped relationship between the number of retail stores and the market size. By allocating more retail stores, the mall can offer a wider variety of products, which makes it more appealing to consumers. This, in turn, leads to a larger market size that benefits all the stores, resulting in the agglomeration effect. However, as the number of retail stores k_R increases, the anchor store's capacity to offer exclusive products (q^M) decreases due to the limited number of commodities ($\bar{k}_0$) offered by the anchor (i.e., the number of monopoly products is $\bar{k}_0 - k_R$). With more retailers, consumers experience a diminishing marginal gain in the utility of having more products A and R, while the marginal loss in utility due to the decreased number of

[8] Fixing k_S at a certain number means we allow for the vacancy of the tenant mix. For example, if there are 5 retail stores ($k_R = 5$) and 3 specialty stores ($k_S = 3$), then there will be 22 vacancies in the tenant mix because we assume the maximum number of stores to be 30.

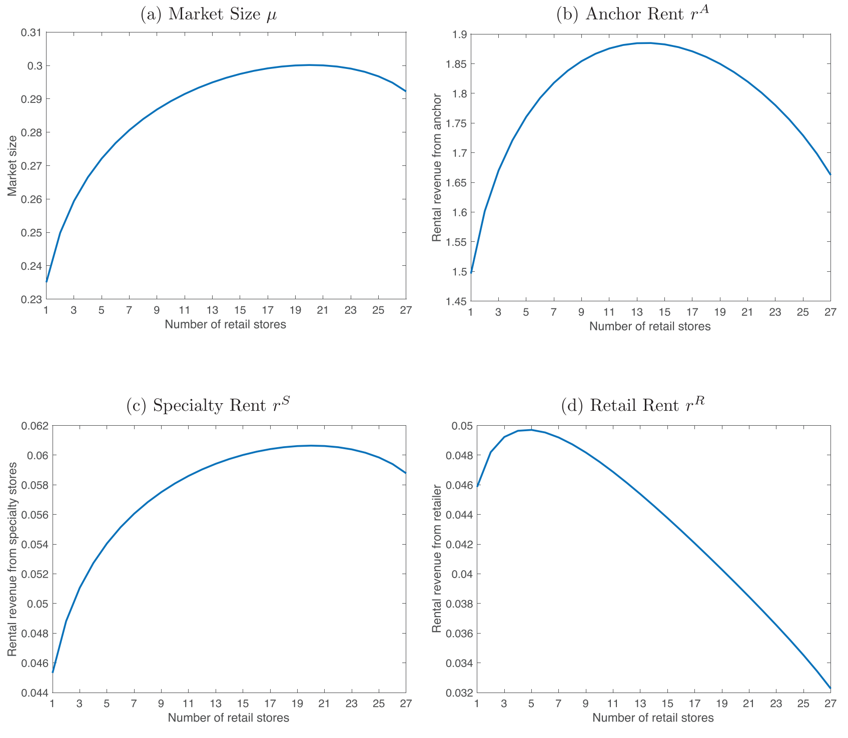


FIGURE 1 The effects of tenanting more retail stores. [Color figure can be viewed at wileyonlinelibrary.com]

exclusive products M increases. Therefore, the market size initially increases as the mall becomes more attractive to consumers, but eventually decreases.

Figure 1b–d shows the rent revenue from the anchor, specialty stores, and retailers, respectively. As discussed in Section 2.4, the total number of consumers (market size) directly affects the profits of all these stores. When more retail stores are allocated, revenues increase initially due to the agglomeration effect, but they decline when too many stores are clustered due to the competition effect. Therefore, we observe a similar pattern as the market size in Figure 1a, particularly in Figure 1c. The hump-shaped pattern is more noticeable for anchor rent in Figure 1b because of the competition between the anchor and retail stores. Rent revenue from retail stores declines rapidly with the number of retailers, as shown in Figure 1d. This is because even a small number of retail stores can cause the competition effect to dominate the agglomeration effect.

Figure 2 depicts the marginal effects of allocating spaces to specialty stores. Similar to Figure 1, we keep the number of retail stores k_R fixed at 3 and vary the number of specialty stores k_S from 1 to 27. In contrast to the case of retail stores, Figure 2a shows a monotonic increase in market size with the number of specialty stores. This is because specialty stores offer experiential services and commodities that complement the anchor and retail stores, resulting in positive externalities that enhance consumer satisfaction. As a consequence, market size increases with more specialty stores. Despite that the competition effect reduces the profits of all specialty stores as shown in Figure 2c, the agglomeration effect benefits the profits of the anchor and retail stores, Figure 2b and d.

Figure 3 shows the optimal tenant mix in the mall. In this analysis, we assume no vacancy in the tenant mix and set the total number of retail and specialty stores equal to the mall's maximum

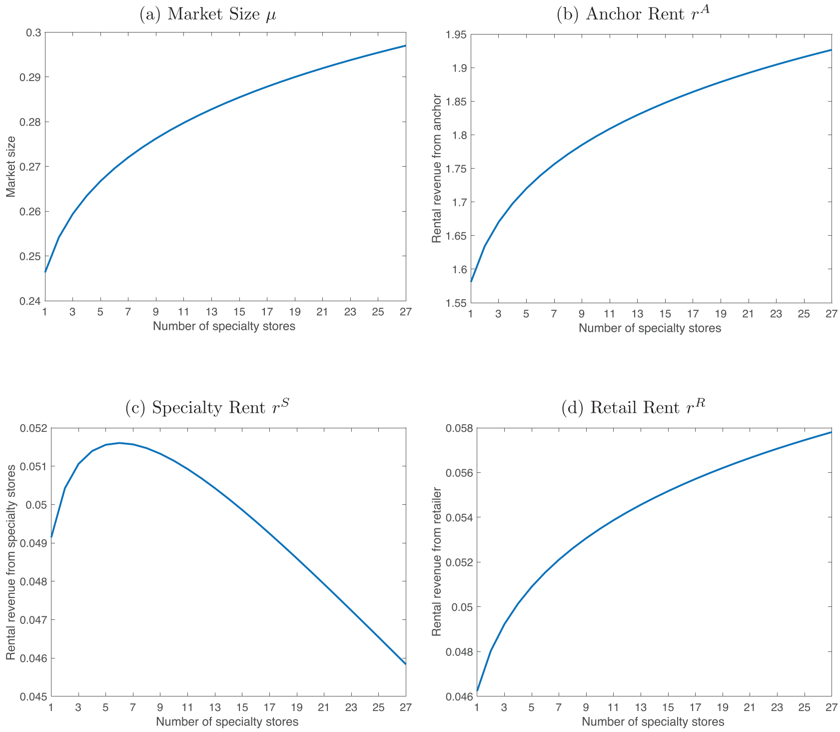


FIGURE 2 The effects of tenanting more specialty stores. [Color figure can be viewed at wileyonlinelibrary.com]

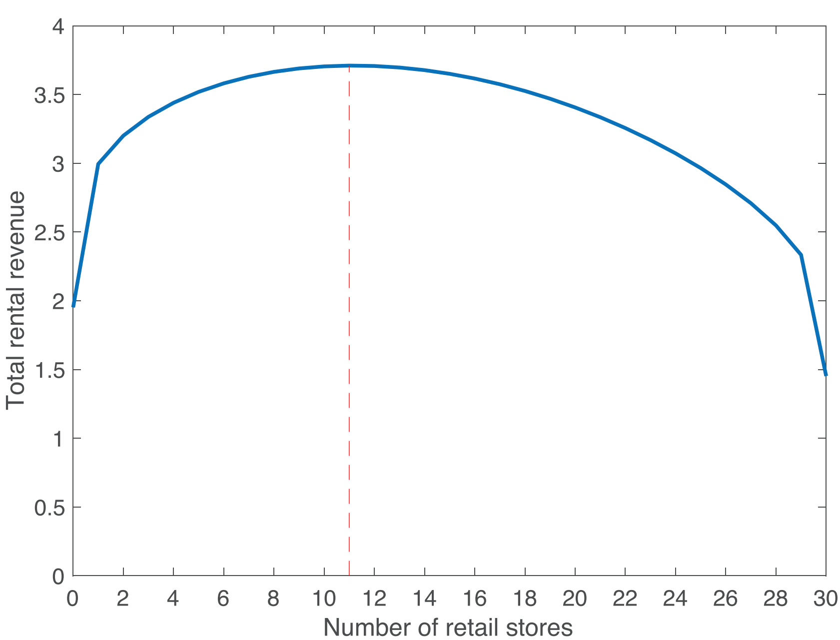


FIGURE 3 Optimal tenant mix. [Color figure can be viewed at wileyonlinelibrary.com]

capacity ($k_R + k_S = \bar{k} = 30$). Given the capacity constraint, $\bar{k}_0$, adding more retail stores implies fewer specialty stores, and the developer must balance this trade-off. The optimal number of retail and specialty stores in equilibrium is 11 and 19, respectively, using the parameters in Table 1.

Moving from left to right on the x-axis shows an increase in the number of retail stores while moving from right to left along the x-axis displays the effect of the number of specialty stores on total rent revenue. By comparing the slopes on two sides from the optimal number, we find that the increase in the developer's profit (total rent revenue) is more significant with the addition of specialty stores. This is due to the complementary role of specialty stores in attracting more consumers, providing more buying options and experiential services, and their commodities not being substitutes for either the anchor store or nonanchor retail stores. Unlike retail stores, the presence of specialty stores does not erode the profits of the anchor store or its traffic-drawing power.

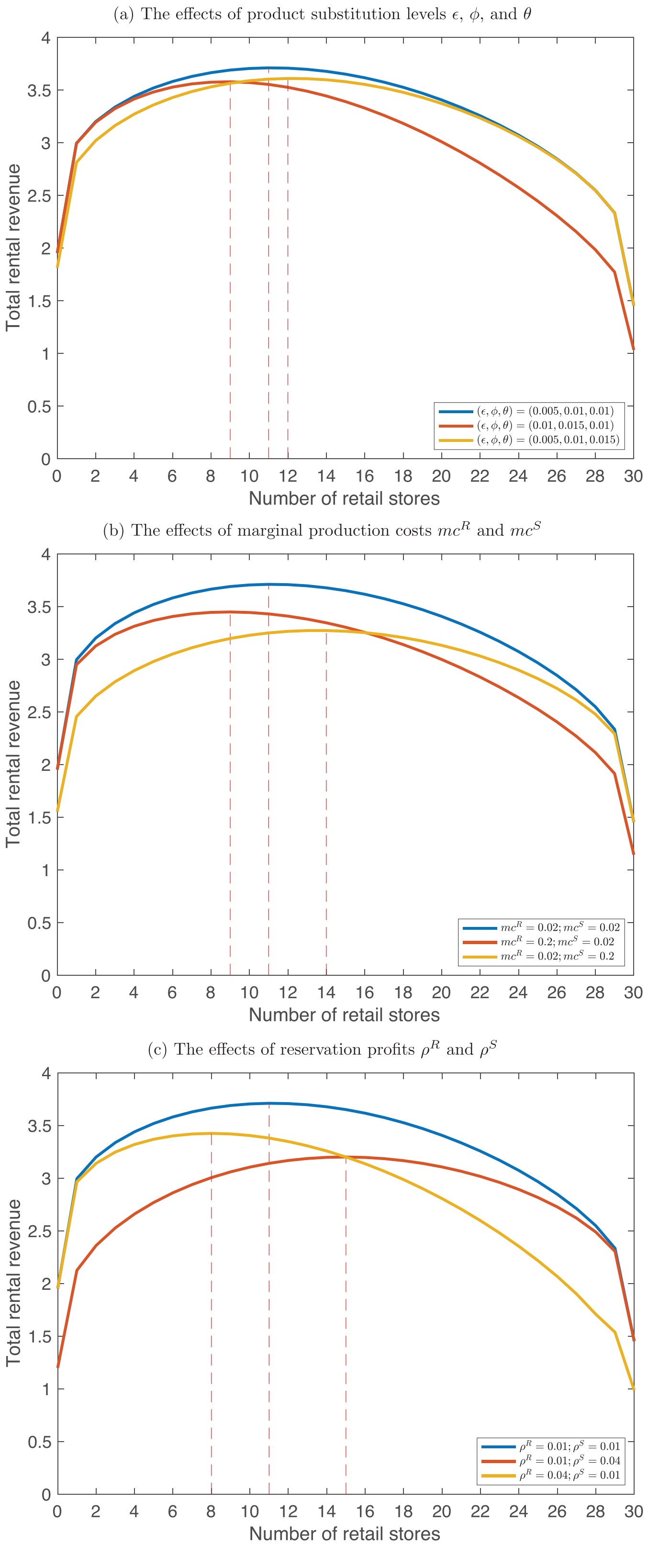
We study two scenarios to evaluate the importance of specialty stores. The first scenario considers a mall that comprises only an anchor store and specialty stores. The second scenario considers a mall with only an anchor store and retail stores. Comparing these two cases reveals that a mall owner achieves higher rent revenue in the first scenario (2 versus 1.5, as shown in Figure 3) due to the absence of competition in the collocation of anchor and specialty stores. Conversely, competition between an anchor store and retail stores substantially diminishes their profits. Therefore, with an anchor store present, a mall comprising only specialty stores is preferred over a mall with only retail stores. As determining the optimal tenant mix depends on various factors, we examine the role of product substitution, marginal production cost, and reservation values in the optimal tenant mix in the following section.

3.2 | Comparative analysis

In the previous section, we examined the trade-off between agglomeration and competition when making an optimal tenant mix decision. Nevertheless, the optimal solution varies depending on various exogenous factors. Therefore, it is crucial for the developer to comprehend the effects of different determinants on the optimal tenant mix. In particular, we focus on three sets of factors: (1) the product substitution levels (ϵ, ϕ, and θ), (2) the marginal production costs of the retail and specialty stores (mc^R and mc^S), and (3) the reservation values of the retail and specialty stores (ρ^R and ρ^S).

Figure 4a shows the effects of product substitutions ϵ, ϕ, and θ on the optimal number of retail stores. The baseline scenario is $\epsilon = 0.005$, $\phi = 0.01$, and $\theta = 0.01$ (blue line), with an optimal number of retail stores of 11. The profits of the anchor and retail stores decrease due to intensified competition when the degrees of substitution for both stores (ϵ and ϕ) are increased by 0.005, as depicted by the red line. Thus, to mitigate this, the developer should allocate fewer retail stores (i.e., the optimal k_R reduces from 11 to 9). The impact of more retail stores is evident from the sharp decline in the red line. The reduction in total revenue enlarges due to the increased values of ϵ and ϕ, which amplify the reduction in rental income from the anchor and retail stores. Similarly, increasing the degree of substitution for specialty stores (θ) by 0.005 (yellow line) results in a smaller number of specialty stores and more retail stores (i.e., the optimal k_R increases from 11 to 12). Both the red line and the orange line are below the blue line because the total rent revenue for the developer is lower with higher product substitution levels. However, the negative effect is smaller for the increased substitution of specialty products. This is because specialty stores offer products or services that do not compete with the anchor or retail stores, resulting in a reduction in rental income from specialty stores only.

FIGURE 4 Effects of product substitution ϵ and ϕ, marginal production costs for commodities sold by the retail stores mc^R and the specialty stores mc^S, and reservation profits of the retail stores ρ^R and the specialty stores ρ^S on optimal tenant mix. [Color figure can be viewed at wileyonlinelibrary.com]



After that, we investigate how the marginal production costs of retail and specialty stores (mc^R and mc^S) affect the optimal tenant mix, with all the other factors held constant. In our model, these costs play a crucial role in determining equilibrium prices, quantities, and market size. Figure 4b displays the effects of increased marginal production costs of mc^R (red line) and mc^S (yellow line) on the optimal number of retail stores. First, compared to the baseline case where $mc^R = 0.02$ and $mc^S = 0.02$ (blue line), when mc^R is raised to 0.2, and mc^S is held constant, the optimal number of retail stores drops (from 11 to 9). The higher marginal production cost reduces profits for retailers, thus decreasing the total rent revenue from them. As a result, the developer replaces retail stores with specialty stores. Second, when mc^R is held constant at 0.02 and mc^S is raised to 0.2, the optimal number of retail stores increases (from 11 to 14). The reason is similar to the previous case: to compensate for the reduced rent revenue from specialty stores, the developer allocates more retail stores.

Lastly, we analyze the impact of reservation values of the retail and specialty stores (ρ^R and ρ^S) on the optimal tenant mix while keeping other factors constant. The reservation value is a measure of how much rent the developer can charge her tenants, and it is influenced by outside profit opportunities. A higher value of ρ implies that locating in other malls is more profitable, leading to the developer having to charge lower rent to keep her tenants inside the mall. Figure 4c illustrates the effects of ρ^S (red line) and ρ^R (yellow line) on the optimal number of retail stores, respectively. As the reservation value of either retail or specialty stores increases, the developer charges less rent from either type of store, resulting in lower total rent revenue. Consequently, both the red and yellow lines are below the blue line (baseline scenario).

To summarize, the results from Figure 4b and c indicate that both marginal production costs and reservation values have comparable effects on the optimal tenant mix. However, changes in these factors from specialty stores have a greater impact on the optimal composition and the rent revenue for developers compared to retail stores. This is due to the fact that increases in marginal production costs and reservation values weaken the complementary role of specialty stores in generating agglomeration economies, affecting the profits of anchor and retail stores as well. Conversely, changes in the product substitution of specialty goods have the least impact on the developer's total rent revenue, since specialty stores offer products that do not compete with those offered by anchor and retailers. As a result, the competition effect only erodes the profits of specialty stores.

4 | CONCLUSION

This article analyzes the optimal tenant composition problem in the current "retail apocalypse." Building on prior models that only consider anchor and nonanchor stores, we introduce a new store type, "specialty stores," which provide experiential consumption. Using a dynamic game model, we analyze the trade-off between agglomeration and competition among the three tenant types. Our findings suggest that increasing the number of specialty stores provides a greater marginal increase in the developer's rent revenue than retail stores due to their complementary role. Specialty stores not only attract more consumers by providing more choices, but their commodities are also not substitutes for anchor and traditional in-line retail stores. Unlike retail stores, the presence of specialty stores does not reduce the profitability of the anchor store or its ability to draw traffic.

In addition, Our analysis also examines how product substitutions, marginal production costs, and reservation values impact the optimal tenant mix and developer's rent revenue. Our findings suggest that both marginal production costs and reservation values have a significant impact on determining the optimal tenant mix. Specifically, changes in either of these factors for specialty stores have a greater effect on the mall owner's rent revenue in the optimal composition compared to retailers. In contrast, the impact of product substitution of specialty goods on the developer's total rent revenue is minimal. This is because specialty stores offer products that do not compete with those sold by the anchor store or nonanchor retailers. Consequently, competition only affects the profits of specialty stores. The implications of our findings on the optimal tenant mix are significant and in line with current trends in the retail industry, which suggest that developers should allocate more space to specialty stores. Our model offers a viable framework to analyze revenue optimization problems in shopping malls, where three types of modern stores coexist in a competitive retail environment. The insights gained from our study can aid developers in making informed decisions about tenant composition and provide valuable guidance for designing successful shopping mall strategies.

Future research may extend our model by allowing more specific consumer preferences. As e-commerce continues to grow, consumers are more likely to visit malls for experiential goods and services rather than traditional anchor stores like department stores or nonanchor retail stores. While our current model does not consider competition between brick-and-mortar stores and online sales, our framework provides a useful foundation for exploring how changing consumer preferences may affect the performance of malls. Further research could build on our analysis by examining these dynamics in more detail.

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APPENDIX A: DERIVATION OF THE DEMAND FUNCTIONS FOR RETAIL PRODUCTS

The inverse demand functions (pricing functions) of retail products are denoted as

$$p_j^A = 1 - q_j^A - \epsilon \sum_{l \neq j}^{k_R} q_l^A - \epsilon \sum_{j}^{k_R} q_j^R$$

$$p_j^R = 1 - q_j^R - \phi \sum_{l \neq j}^{k_R} q_l^R - \phi \sum_{j}^{k_R} q_j^A.$$

We expand the system of inverse demand functions and rearrange the terms

$$\begin{aligned}
p_1^A &= 1 - q_1^A - \epsilon q_2^A - \dots - \epsilon q_{k_R}^A - \epsilon q_1^R - \epsilon q_2^R - \dots - \epsilon q_{k_R}^R \\
p_2^A &= 1 - \epsilon q_1^A - q_2^A - \dots - \epsilon q_{k_R}^A - \epsilon q_1^R - \epsilon q_2^R - \dots - \epsilon q_{k_R}^R \\
&\vdots \\
p_{k_R}^A &= 1 - \epsilon q_1^A - \epsilon q_2^A - \dots - q_{k_R}^A - \epsilon q_1^R - \epsilon q_2^R - \dots - \epsilon q_{k_R}^R
\end{aligned}$$

$$
\begin{aligned}
p_1^R &= 1 - \phi q_1^A - \phi q_2^A - \ldots - \phi q_{k_R}^A - q_1^R - \phi q_2^R - \ldots - \phi q_{k_R}^R \\
p_2^R &= 1 - \phi q_1^A - \phi q_2^A - \ldots - \phi q_{k_R}^A - \phi q_1^R - q_2^R - \ldots - \phi q_{k_R}^R \\
&\vdots \\
p_{k_R}^R &= 1 - \phi q_1^A - \phi q_2^A - \ldots - \phi q_{k_R}^A - \phi q_1^R - \phi q_2^R - \ldots - q_{k_R}^R.
\end{aligned}
$$

We can represent the system of inverse demand functions into a matrix form

$$\mathbf{p} = 1 - \mathbf{A}\mathbf{q}$$

$$
\text{where} \quad \mathbf{p} = \begin{bmatrix} p_1^A \\ \vdots \\ p_{k_R}^A \\ p_1^R \\ \vdots \\ p_{k_R}^R \end{bmatrix}; \quad \mathbf{q} = \begin{bmatrix} q_1^A \\ \vdots \\ q_{k_R}^A \\ q_1^R \\ \vdots \\ q_{k_R}^R \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} 1 & \epsilon & \cdots & \epsilon & \epsilon & \epsilon & \cdots & \epsilon \\ \epsilon & 1 & \cdots & \epsilon & \epsilon & \epsilon & \cdots & \epsilon \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \epsilon & \epsilon & \cdots & 1 & \epsilon & \epsilon & \cdots & \epsilon \\ \phi & \phi & \cdots & \phi & 1 & \phi & \cdots & \phi \\ \phi & \phi & \cdots & \phi & \phi & 1 & \cdots & \phi \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi & \phi & \cdots & \phi & \phi & \phi & \cdots & 1 \end{bmatrix}.
$$

To represent the system of demand functions such that $\mathbf{q} = \mathbf{A}^{-1}(1 - \mathbf{p})$, we can find the inverse of the matrix $\mathbf{A}$ by Sherman-Morrison formula. This formula states that suppose $B \in \mathbb{R}^{n\times n}$ is an invertible square matrix and $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ are column vectors. Then $\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T}$ is invertible if and only if $1 + \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u} \neq 0$. In this case,

$$(\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T})^{-1} = \mathbf{B}^{-1} - \frac{\mathbf{B}^{-1}\mathbf{u} \cdot \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}}{\mathbf{1} + \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u}}.$$

By using this formula, we have to decompose the matrix $\mathbf{A}$ into the form of $\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T}$ such that

$$
\mathbf{A} = \begin{bmatrix} 1 & \epsilon & \cdots & \epsilon & \epsilon & \epsilon & \cdots & \epsilon \\ \epsilon & 1 & \cdots & \epsilon & \epsilon & \epsilon & \cdots & \epsilon \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \epsilon & \epsilon & \cdots & 1 & \epsilon & \epsilon & \cdots & \epsilon \\ \phi & \phi & \cdots & \phi & 1 & \phi & \cdots & \phi \\ \phi & \phi & \cdots & \phi & \phi & 1 & \cdots & \phi \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi & \phi & \cdots & \phi & \phi & \phi & \cdots & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1-\epsilon & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1-\epsilon & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1-\epsilon & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1-\phi & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1-\phi & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1-\phi \end{bmatrix}}_{\mathbf{B}} + \underbrace{\begin{bmatrix} \epsilon \\ \epsilon \\ \vdots \\ \epsilon \\ \phi \\ \phi \\ \vdots \\ \phi \end{bmatrix}}_{\mathbf{u}} \underbrace{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}_{\mathbf{v}^\mathbf{T}}.
$$

The matrix $\mathbf{A}$ can be expressed as $\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T}$ where $B \in \mathbb{R}^{2k_R\times 2k_R}$ is an invertible square matrix, $\mathbf{u}, \mathbf{v} \in \mathbb{R}^{2k_R}$ are column vectors, and $1 + \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u} \neq 0$. Therefore, we can derive $\mathbf{A}^{-1} = (\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T})^{-1} = \mathbf{B}^{-1} - \frac{\mathbf{B}^{-1}\mathbf{u}\cdot\mathbf{v}^\mathbf{T}\mathbf{B}^{-1}}{1+\mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u}}$.

First of all, we derive the term of $\mathbf{B^{-1}u \cdot v^T B^{-1}}$:

$$\mathbf{B^{-1}u \cdot v^T B^{-1}} = \begin{bmatrix} \frac{\epsilon}{(1-\epsilon)^2} & \cdots & \frac{\epsilon}{(1-\epsilon)^2} & \frac{\epsilon}{(1-\epsilon)(1-\phi)} & \cdots & \frac{\epsilon}{(1-\epsilon)(1-\phi)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\epsilon}{(1-\epsilon)^2} & \cdots & \frac{\epsilon}{(1-\epsilon)^2} & \frac{\epsilon}{(1-\epsilon)(1-\phi)} & \cdots & \frac{\epsilon}{(1-\epsilon)(1-\phi)} \\ \frac{\phi}{(1-\epsilon)(1-\phi)} & \cdots & \frac{\phi}{(1-\epsilon)(1-\phi)} & \frac{\phi}{(1-\phi)^2} & \cdots & \frac{\phi}{(1-\phi)^2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\phi}{(1-\epsilon)(1-\phi)} & \cdots & \frac{\phi}{(1-\epsilon)(1-\phi)} & \frac{\phi}{(1-\phi)^2} & \cdots & \frac{\phi}{(1-\phi)^2} \end{bmatrix}.$$

Second, we derive the term of $\mathbf{1 + v^T B^{-1} u}$:

$$\mathbf{1 + v^T B^{-1} u} = 1 + k_R\left(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi}\right).$$

As a result, we can derive A^{-1} :

$$\mathbf{A}^{-1} = B^{-1} - \frac{B^{-1}u \cdot v^T B^{-1}}{1 + v^T B^{-1} u}$$

$$= \begin{bmatrix} \frac{1}{1-\epsilon} - \frac{\epsilon}{(1-\epsilon)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\epsilon}{(1-\epsilon)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \frac{-\epsilon}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\epsilon}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-\epsilon}{(1-\epsilon)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{1}{1-\epsilon} - \frac{\epsilon}{(1-\epsilon)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \frac{-\epsilon}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\epsilon}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} \\ \frac{-\phi}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\phi}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \frac{1}{1-\phi} - \frac{\phi}{(1-\phi)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\phi}{(1-\phi)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-\phi}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{-\phi}{(1-\epsilon)(1-\phi)[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \frac{-\phi}{(1-\phi)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} & \cdots & \frac{1}{1-\phi} - \frac{\phi}{(1-\phi)^2[1+k_R(\frac{\epsilon}{1-\epsilon}+\frac{\phi}{1-\phi})]} \end{bmatrix}.$$

Demand function $\mathbf{q} = \mathbf{A}^{-1}(1 - \mathbf{p})$ can be expressed as

$$q_j^A = \frac{1}{1-\epsilon}\left[1 - \frac{\epsilon}{1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})}\left(k_R(\frac{1}{1-\epsilon} + \frac{1}{1-\phi})\right)\right] - (\frac{1}{1-\epsilon})(1 - \frac{\epsilon}{(1-\epsilon)[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]})p_j^A$$

$$+ \frac{\epsilon}{(1-\epsilon)^2[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]}\sum_{l \neq j}^{k_R} p_l^A + \frac{\epsilon}{(1-\epsilon)(1-\phi)[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]}\sum_{j}^{k_R} p_j^R$$

$$q_j^R = \frac{1}{1-\phi}\left[1 - \frac{\phi}{[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]}\left(k_R(\frac{1}{1-\epsilon} + \frac{1}{1-\phi})\right)\right] - (\frac{1}{1-\phi})(1 - \frac{\phi}{(1-\phi)[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]})p_j^R$$

$$+ \frac{\phi}{(1-\phi)^2[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]}\sum_{l \neq j}^{k_R} p_l^R + \frac{\phi}{(1-\epsilon)(1-\phi)[1 + k_R(\frac{\epsilon}{1-\epsilon} + \frac{\phi}{1-\phi})]}\sum_{j}^{k_R} p_j^A.$$

APPENDIX B: DERIVATION OF THE DEMAND FUNCTIONS FOR THE SPECIALTY PRODUCTS

The inverse demand functions (pricing functions) of specialty products are denoted as

$$p_z^S = 1 - q_z^S - \theta \sum_{l \neq z}^{k_S} q_l^S.$$

We expand the system of inverse demand functions and rearrange the terms

$$\begin{aligned} p_1^S &= 1 - q_1^S - \theta q_2^S - \dots - \theta q_{k_S}^S \\ p_2^S &= 1 - \theta q_1^S - q_2^S - \dots - \theta q_{k_S}^S \\ &\vdots \\ p_{k_S}^S &= 1 - \theta q_1^S - \theta q_2^S - \dots - q_{k_S}^S. \end{aligned}$$

We can represent the system of inverse demand functions into a matrix form

$$\mathbf{p} = 1 - \mathbf{A}\mathbf{q}$$

where $\quad \mathbf{p} = \begin{bmatrix} p_1^S \\ \vdots \\ p_{k_S}^S \end{bmatrix}; \quad \mathbf{q} = \begin{bmatrix} q_1^S \\ \vdots \\ q_{k_S}^S \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} 1 & \theta & \dots & \theta \\ \theta & 1 & \theta & \vdots \\ \vdots & \theta & \ddots & \theta \\ \theta & \dots & \theta & 1 \end{bmatrix}.$

To represent the system of demand functions such that $\mathbf{q} = \mathbf{A}^{-1}(1 - \mathbf{p})$, we can find the inverse of the matrix $\mathbf{A}$ by Sherman-Morrison formula. By using this formula, we have to decompose the matrix $\mathbf{A}$ into the form of $\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T}$ such that

$$\mathbf{A} = \begin{bmatrix} 1 & \theta & \dots & \theta \\ \theta & 1 & \dots & \theta \\ \vdots & \theta & \ddots & \theta \\ \theta & \theta & \dots & 1 \end{bmatrix} = \underbrace{(1-\theta)\mathbf{I}}_{\mathbf{B}} + \underbrace{\theta \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{u}} \underbrace{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}_{\mathbf{v}^\mathbf{T}}.$$

The matrix $\mathbf{A}$ can be expressed as $\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T}$ where $B \in \mathbb{R}^{k_S \times k_S}$ is an invertible square matrix, $\mathbf{u}, \mathbf{v} \in \mathbb{R}^{k_S}$ are column vectors and $1 + \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u} \neq 0$. Therefore, we can derive $\mathbf{A}^{-1} = (\mathbf{B} + \mathbf{u} \cdot \mathbf{v}^\mathbf{T})^{-1} = \mathbf{B}^{-1} - \frac{\mathbf{B}^{-1}\mathbf{u} \cdot \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}}{1 + \mathbf{v}^\mathbf{T}\mathbf{B}^{-1}\mathbf{u}}$. First of all, we derive the term of $B^{-1}u \cdot v^T B^{-1}$:

$$\begin{aligned} B^{-1}u \cdot v^T B^{-1} &= \left(\frac{1}{1-\theta} \mathbf{I}_{k_S \times k_S} \right) \left(\epsilon_{k_S \times 1} \right) \left(\mathbf{1}_{1 \times k_S} \right) \left(\frac{1}{1-\theta} \mathbf{I}_{k_S \times k_S} \right) \\ &= \frac{\theta}{(1-\theta)^2} \mathbf{1}_{k_S \times k_S}. \end{aligned}$$

Second, we derive the term of $1 + v^T B^{-1} u$:

$$1 + v^T B^{-1} u = 1 + \left(\mathbf{1}_{1 \times k_S} \right) \left(\frac{1}{1-\theta} \mathbf{I}_{k_S \times k_S} \right) \left(\theta_{k_S \times 1} \right)$$

$$= 1 + \frac{\theta}{1-\theta}\mathbf{1}_{1\times k_S} \cdot \mathbf{1}_{k_S\times 1}$$

$$= 1 + k_S\frac{\theta}{1-\theta}.$$

As a result, we can derive A^{-1}:

$$\begin{aligned}
A^{-1} &= B^{-1} - \frac{B^{-1}u \cdot v^T B^{-1}}{1 + v^T B^{-1} u} \\
&= \frac{1}{1-\theta}\mathbf{I}_{k_S\times k_S} - \frac{\frac{\theta}{(1-\theta)^2}\mathbf{1}_{k_S\times k-2}}{1 + k_S\frac{\theta}{1-\theta}} \\
&= \frac{1}{1-\theta}\left[\mathbf{I}_{k_S\times k_S} - \frac{\theta}{1-\theta+k_S\theta}\mathbf{1}_{k_S\times k_S}\right] \\
&= \frac{1}{1-\theta}\begin{bmatrix} 1-\frac{\theta}{1-\theta+k_S\theta} & -\frac{\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ -\frac{\theta}{1-\theta+k_S\theta} & 1-\frac{\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\theta}{1-\theta+k_S\theta} & \dots & -\frac{\theta}{1-\theta+k_S\theta} & 1-\frac{\theta}{1-\theta+k_S\theta} \end{bmatrix}_{k_S\times k_S} \\
&= \frac{1}{1-\theta}\begin{bmatrix} \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} & -\frac{\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ -\frac{\theta}{1-\theta+k_S\theta} & \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\theta}{1-\theta+k_S\theta} & \dots & -\frac{\theta}{1-\theta+k_S\theta} & \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} \end{bmatrix}_{k_S\times k_S}.
\end{aligned}$$

Demand function $\mathbf{q} = \mathbf{A}^{-1}(1 - \mathbf{p})$ can be expressed as

$$\mathbf{q} = A^{-1}(\mathbf{1} - \mathbf{p})$$

$$\begin{bmatrix} q_1^S \\ \vdots \\ q_{k_S}^S \end{bmatrix} = \frac{1}{1-\theta}\begin{bmatrix} \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} & -\frac{\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ -\frac{\theta}{1-\theta+k_S\theta} & \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} & \cdots & -\frac{\theta}{1-\theta+k_S\theta} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\theta}{1-\theta+k_S\theta} & -\frac{\theta}{1-\theta+k_S\theta} & \cdots & \frac{1+(k_S-2)\theta}{1-\theta+k_S\theta} \end{bmatrix}_{k_S\times k_S} \begin{bmatrix} 1-p_1^S \\ \vdots \\ 1-p_{k_S}^S \end{bmatrix}$$

$$q_z^S = \frac{1}{1+(k_S-1)\theta} - \frac{1+(k_S-2)\theta}{(1-\theta)(1+(k_S-1)\theta)}p_z^S + \frac{\theta}{(1-\theta)(1+(k_S-1)\theta)}\sum_{l\neq z}^{k_S} p_l^S.$$

APPENDIX C: DERIVATION OF THE EFFECTS OF K_R AND K_S ON DEVELOPER'S PROFIT:

$$\begin{aligned}\Pi^D(k_R,k_S) &= r^A(k_R,k_S)+\sum_{j=1}^{k_R} r_j^R(k_R,k_S)+\sum_{z=1}^{k_S} r_z^S(k_R,k_S)\\ &= \left[\mu(k_R,k_S)\pi^A(k_R)-\rho^A\right]+\sum_{j=1}^{k_R}\left[\mu(k_R,k_S)\pi^R(k_R)-\rho^R\right]+\sum_{z=1}^{k_S}\left[\mu(k_R,k_S)\pi^S(k_S)-\rho^S\right]\\ &= \left[\mu(k_R,k_S)\pi^A(k_R)-\rho^A\right]+k_R\left[\mu(k_R,k_S)\pi^R(k_R)-\rho^R\right]+k_S\left[\mu(k_R,k_S)\pi^S(k_S)-\rho^S\right].\end{aligned}$$

Effects of k_R on rent payment on different stores:

$$\frac{d}{dk_R}r^A(k_R,k_S)=\pi^A\frac{d\mu}{dk_R}+\mu\frac{d\pi^A}{dk_R},$$

$$\frac{d}{dk_R}\sum_{j=1}^{k_R} r_j^R(k_R,k_S)=k_R\left(\pi^R\frac{d\mu}{dk_R}+\mu\frac{d\pi^R}{dk_R}\right)+r^R,$$

$$\frac{d}{dk_R}\sum_{z=1}^{k_S} r_z^S(k_R,k_S)=(\bar{k}-k_R)\left(\pi^S\frac{d\mu}{dk_R}+\mu\frac{d\pi^S}{dk_R}\right)-r^S.$$

Effects of k_S on rent payment on different stores:

$$\frac{d}{dk_S}r^A(k_R,k_S)=\pi^A\frac{d\mu}{dk_S}+\mu\frac{d\pi^A}{dk_S},$$

$$\frac{d}{dk_S}\sum_{j=1}^{k_R} r_j^R(k_R,k_S)=(\bar{k}-k_S)\left(\pi^R\frac{d\mu}{dk_S}+\mu\frac{d\pi^R}{dk_S}\right)-r^R,$$

$$\frac{d}{dk_S}\sum_{z=1}^{k_S} r_z^S(k_R,k_S)=k_S\left(\pi^S\frac{d\mu}{dk_S}+\mu\frac{d\pi^S}{dk_S}\right)+r^S.$$

Total effects of k_R on developer's profit:

$$\begin{aligned}\frac{d\Pi^D}{dk_R} &= \frac{d}{dk_R}r^A(k_R,k_S)+\frac{d}{dk_R}\sum_{j=1}^{k_R} r_j^R(k_R,k_S)+\frac{d}{dk_R}\sum_{z=1}^{k_S} r_z^S(k_R,k_S)\\ &= \underbrace{r^R}_{\text{rent income}}+\underbrace{\left[\pi^A+k_R\pi^R+(\bar{k}-k_R)\pi^S\right]\frac{d\mu}{dk_R}}_{\text{agglomeration effect}}+\underbrace{\mu\left(\frac{d\pi^A}{dk_R}+k_R\frac{d\pi^R}{dk_R}+(\bar{k}-k_R)\frac{d\pi^S}{dk_R}\right)}_{\text{competition effect}}-\underbrace{r^S}_{\text{opportunity cost}}.\end{aligned}$$

Total effects of k_S on developer's profit:

$$\begin{aligned}\frac{d\Pi^D}{dk_S} &= \frac{d}{dk_S}r^A(k_R,k_S)+\frac{d}{dk_S}\sum_{j=1}^{k_R} r_j^R(k_R,k_S)+\frac{d}{dk_S}\sum_{z=1}^{k_S} r_z^S(k_R,k_S)\\ &= \underbrace{r^S}_{\text{rent income}}+\underbrace{\left[\pi^A+(\bar{k}-k_S)\pi^R+k_S\pi^S\right]}_{\text{agglomeration effect}}\frac{d\mu}{dk_S}+\underbrace{\mu\left[\frac{d\pi^A}{dk_S}+(\bar{k}-k_S)\frac{d\pi^R}{dk_S}+k_S\frac{d\pi^S}{dk_S}\right]}_{\text{competition effect}}-\underbrace{r^R}_{\text{opportunity cost}}.\end{aligned}$$